TechNotes March 1993

The ComPort Computing Company

12230 Palmfree St., Houston, Texas 77034, U. S. A.

1-713-947-3363

Email: wfair@comportco.com

THE PARAMETER C_De^{2S}

W. B. Fair, Jr.

It has been noted that the parameter C_De^{2S} can be used to correlate type curves for wells with wellbore storage and skin. This note presents the mathematical derivation of the parameter, shows when it is valid and explains the implication for well test analysis.

Mathematical Background

The Laplace transform of the general solution[1] for wellbore pressure with radial flow, wellbore storage and skin is shown in Equation 1.

$$\bar{p}_{wD}=\frac{1}{s}\frac{s\,\bar{p}_D+S}{1+C_D s(s\,\bar{p}_D+S)}$$
$$where\ \ \bar{p}_D=\frac{K_0(\sqrt{s})}{s\sqrt{s}K_1(\sqrt{s})} \qquad (1)$$

This equation can be transformed to a new time variable defined by $\tau=t_D/C_D$ with solution shown in Equation 2. Note that both relations are identical; only the time variable has been changed, but the solution no longer has an explicit storage parameter, C_D. The only place where both C_D and S occur is in the term $s\,\bar{p}_D+S$.

$$\bar{p}_{wD}=\frac{1}{s}\frac{s\,\bar{p}_D+S}{1+s(s\,\bar{p}_D+S)}$$
$$where\ \ \bar{p}_D=\frac{K_0(\sqrt{s/C_D})}{s\sqrt{s/C_D}K_1(\sqrt{s/C_D})} \qquad (2)$$

Note that at long times, $s\rightarrow 0$ and $\sqrt{s/C_D}K_1(\sqrt{s/C_D})\rightarrow 1$ and $K_0(\sqrt{s})\rightarrow -\ln(\frac{\sqrt{s}}{2})-\gamma$.

Substituting and rearranging yields Equation 3, where both C_D and S are contained in only 1 term as the parameter C_De^{2S}.

$$s\,\bar{p}_D + S \rightarrow -\left[\ln\left(\frac{1}{2}\sqrt{\frac{s}{C_D}}\right) - S + \gamma\right] = -\left[\ln\left(\frac{1}{2}\sqrt{s}\right) - \frac{1}{2}\ln\left(C_D e^{2S}\right) + \gamma\right] \quad \text{......(3)}$$

In view of Equation 3, the use of C_De^{2S} as a correlating parameter is valid when the long time approximation to the line source solution is valid, which is most of the time. Note however, that this also indicates that since a single parameter is present, it may be difficult to differentiate between the curves in terms of storage and skin. Any combination of storage and skin yielding the same value of C_De^{2S} will have the same overall curve shape.

At first glance it would appear that by using C_De^{2S} we have lost the ability to determine both C_D and S, but that is not true. Since C_De^{2S} is a correlating parameter and there is no loss of information, both C_D and S can be recovered from the log-log type curve match. The procedure is as follows:

1) From the type curve match, determine the pressure and time match points and the value of C_De^{2S}.
2) Calculate kh/μ using the reservoir and well data: $kh/\mu = (qB)[p_D/\Delta p]$
3) Using the time match point and the value of kh/μ, calculate the storage constant C from $C = 2\pi(kh/\mu)[t/(t_D/C_D)]$ and also $C_D = C/(2\pi\phi h c_t r_w^2)$.
4) Calculate the skin from $S = ½ \ln(C_De^{2S}/C_D)$

Nomenclature

C_D	wellbore storage parameter
C_α	apparent wellbore storage parameter
p_D	phase redistibution pressure
p_{wD}	bottomhole pressure
S	Skin factor
s	Laplace transform independent variable
K_0, K_1	Modified Bessel functions second kind
γ	Euler's constant = 0.5772 ...

References

1. Agarwal, R. G., Al-Hussainy, Rafi, and Ramey, H. J. Jr., “An Investigation of Well Bore Storage and Skin Effect in Unsteady Liquid Flow: I. Analytical Treatment,” SPEJ (Sept 1970) 279-290, Trans. AIME, 249..