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**SOME IMPLICATIONS OF P<sub>φ</sub> ON PHASE SEGREGATION PHENOMENA IN WELLS**

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In the 1981 paper on wellbore phase redistribution<sup>1</sup>, a "phase redistribution" pressure function was defined to explain the difference in pressures observed in wells under going phase segregation. Later it was shown that this pressure actually represents the pressure change at constant wellbore mass<sup>2</sup>. In the original paper, an exponential function was used for the pressure, however, later investigators seemed to have better results using an error function form<sup>3</sup>. Although not thoroughly documented in the original 1981 paper, the choice of an exponential function was based mainly on the ability to represent field data. The scant amount of laboratory data available was too consistent and controlled to directly model actual wellbore conditions. The laboratory data seemed to show a somewhat exponential form at early times, but all the laboratory and simulated data to date seem to show that the phase redistribution effect dies at a given time. In contrast, all attempts to use such a model for the pressure function yield a discontinuity not observed in field data. For that reason, the choice of an exponential phase redistribution pressure function has seemed to give an overall acceptable representation of field data.

However, there has always been the nagging question of what the pressure function should really be and what parameters control the extent of the phase redistribution in both pressure level and time. For that reason, this work evaluates the effect of the choice of an exponential or other functional form for phase redistribution pressure on the physical parameters of a fluid system. It is assumed that the fluid system of interest consists of a liquid column through which gas is flowing. It is also assumed that the wellbore is closed at the surface, so that the gas will collect to form a gas cap within the wellbore. For simplicity, isothermal conditions are assumed, although this assumption can be removed as shown in Reference 2.

To evaluate the performance of a wellbore under going phase redistribution, we can write the relation for the mass of gas collecting near the surface. Since the mass is equal to the gas volume multiplied by the gas density and equal to the rate of gas influx from the underlying liquid column, the time derivative of mass can be written as Equation 1. (Downward flow or injection is positive, so the gas flowing toward the surface would be a negative rate.)

$$\frac{dm_{gs}}{dt} = -q_{gL}\rho_{gL} = \rho_{gs} \frac{dV_{gs}}{dt} + V_{gs} \frac{d\rho_{gs}}{dt} \dots\dots\dots (1)$$

Solving Equation 1 for the gas density derivative yields Equation 2.

$$\frac{d\rho_{gs}}{dt} = \frac{q_{gL}\rho_{gL}}{V_{gs}} - \frac{\rho_{gs}}{V_{gs}} \frac{dV_{gs}}{dt} \dots\dots\dots (2)$$

Assuming the gas density depends only on pressure, the gas density can be written as Equation 3.

$$\rho_g = \beta p$$

$$\beta = \frac{2.7\gamma_g}{zT} \approx \text{constant} \dots\dots\dots(3)$$

Substituting the gas density relation into Equation 2 and assuming that the surface pressure is indicative of the gas density yields Equation 4, which expresses the change in surface gas pressure in terms of influx from the liquid column and the volume change in the gas near the surface.

$$\frac{dp_s}{dt} = \frac{dp_\phi}{dt} = -\frac{q_{gL}P_L}{V_{gs}} - \frac{p_s}{V_{gs}} \frac{dV_{gs}}{dt} \dots\dots\dots(4)$$

Note that we have assumed that no liquid flows into the well, so the pressure change at the surface is the pressure change at constant wellbore mass<sup>2</sup>, which is identical to the phase redistribution pressure. Therefore, by specifying how the gas influx rate and the volume change occur, the  $p_\phi$  function can be evaluated.

**Exponential Form**

It is easy to show that if the first term on the right in Equation 4 is constant and the second term is proportional to pressure, the differential equation can be solved, giving an exponential form for the phase redistribution pressure. To show this we write Equation 4 as Equation 5. The solution is given in Equation 6, where  $p_0$  is the initial surface pressure and  $p_{max}$  is the final surface pressure at infinite time.

$$\frac{dp_s}{dt} = a - bp_s \dots\dots\dots(5)$$

$$p_s = p_{max} - (p_{max} - p_0)e^{-bt} \dots\dots\dots(6)$$

Noting that the phase redistribution function is the difference in the surface pressure allows us to express the phase redistribution pressure function as Equation 7, which is the same form as hypothesized earlier<sup>1</sup>.

$$p_\phi = (p_{max} - p_0)(1 - e^{-bt}) = C_\phi \left(1 - e^{-t/\alpha}\right) \dots\dots\dots(7)$$

This expression also allows us to determine the significance of the parameters in the exponential phase redistribution function. First, we note that  $C_\phi$  is the maximum pressure change, which is consistent with previous observations. Moreover,  $C_\phi$  arises from the long time condition that the pressure must approach a constant value. This value is the ratio of  $a/b$  in Equation 5, which specifies that the rate of mass influx divided by the rate of volume change is a constant. This is illustrated in Equation 8.

$$C_\phi = p_{max} - p_0$$

$$p_{max} = \frac{a}{b} = \frac{-q_{gL}P_L}{\frac{dV_{gs}}{dt}} \dots\dots\dots(8)$$

In addition, the parameter  $\alpha$  in the phase redistribution function can be seen to be  $1/b$  in Equation 5. This can be expressed in terms of physical parameters as shown in Equation 9.

$$\frac{1}{\alpha} = b = \frac{1}{V_{gs}} \frac{dV_{gs}}{dt} \dots\dots\dots(9)$$

Therefore, under this interpretation, the time parameter in the exponential phase redistribution pressure function is equal to the reciprocal of the relative rate of volume change of the gas near the surface.

Moreover, in a vertical well with constant cross section, the volume of gas is proportional to the fluid level. Therefore, the time parameter is also related to the relative rate of change in the fluid level.

We now have an understanding of the assumptions that lead to the exponential form for the phase redistribution pressure. Additionally, we also have a means to relate the parameters to physical properties. It is possible, then, to evaluate the implications of the assumptions and investigate other forms for the pressure function.

First, considering the expression for  $p_{max}$ , which is related to  $C_\phi$ , we have assumed that the rate of gas volume increase, is directly proportional to the mass rate of gas influx. Assuming a constant wellbore cross section, the fluid level change can be expressed as Equation 10, where  $C_1$  is a constant and  $A$  represents the cross sectional area. Note the negative sign, which indicates that the gas is flowing upward while the fluid level is increasing or moving downward.

$$\frac{dx_L}{dt} = -\frac{C_1 q_{gL} \rho_{gL}}{A} \dots\dots\dots (10)$$

Noting that the volumetric gas flow rate is equal to the relative gas superficial velocity at the top of the liquid column times the area gives Equation 11.

$$\frac{dx_L}{dt} = -C_1 (v_{sg} - v_{sL}) \rho_{gL} \dots\dots\dots (11)$$

It is apparent that the superficial liquid velocity at the interface must be equal to the velocity of the interface, so substituting and solving for the superficial gas velocity yields Equation 12. The relative gas velocity can also be expressed in Equation 13.

$$v_{sg} = v_{sL} \left( 1 - \frac{1}{C_1 \rho_{gL}} \right) \dots\dots\dots (12)$$

$$v_{sg} - v_{sL} = -\frac{v_{sL}}{C_1 \rho_{gL}} \dots\dots\dots (13)$$

If we can neglect changes in the gas density, then the relative gas velocity will be proportional to the liquid velocity. If the gas density increases due to increasing pressure, then the relative gas velocity will decrease relative to the liquid velocity.

Although beyond the scope of this note, it is also possible to show that the relationship between the gas and liquid velocities implies a specific gas void fraction, or conversely, liquid hold up. (For example, see Reference 4.) Unfortunately, since we are modeling transient flow behavior, it is not apparent the steady state correlations can be applied. In addition, the authors of most correlations state that transient flow effects were apparent in the experimental setups used to measure the liquid holdup fraction. This was apparent from the observation that measurements could not be taken too close to the fluid inlet.

The evaluation of the time parameter,  $\alpha$ , is much easier. Writing Equation 9 in terms of the fluid level gives Equation 14.

$$\frac{1}{\alpha} = \frac{1}{x_L} \frac{dx_L}{dt} \dots\dots\dots (14)$$

This equation has a direct solution, which states that the fluid level will have an exponential character, as shown in Equation 15.

$$x_L = x_{L0} e^{t/\alpha} \dots\dots\dots (15)$$

Note that this representation presents a problem, since if the initial fluid level is zero (at the surface), the fluid level can never change. In practice, there will always be some gas at the surface, so even though  $x_{L0}$  may be small, an exponential change will occur.

This, however, presents another problem, since we know that for a closed system, a stable static fluid level must occur. Equation 15 implies that a stable fluid level cannot occur, since  $x_L$  becomes infinitely large as

time increases. The solution to the paradox is actually quite simple. In many wells it has been observed that gas continues to flow into the wellbore after shut in. In many cases, the fluid level increases until all of the liquid is pushed back into the reservoir and the entire wellbore is filled with gas. Under these conditions, a continuously increasing fluid level is a possibility up to the point where there is no liquid. Normally, due to convolution with the reservoir pressure response, wellbore effects die out before that point is reached and the long time behavior of the fluid level is unimportant.

Therefore, based on the hypothetical analysis presented here, it appears that the exponential form for the phase redistribution pressure function is most likely a limiting form for a long wellbore when free gas is flowing into the wellbore and the gas flow continues long after the well is shut-in. Interestingly, all of the wells evaluated in conjunction with Reference 1 approximately satisfied this criteria, since they were producing well below the bubble point pressure.

**Error Function Form**

Following the same procedure, it is possible to investigate the implications of the error function form for phase redistribution proposed by Hegeman, et. al.<sup>3</sup> It can be shown that the error function form arises by solution of an equation of the form of Equation 16, so the appropriate relationships can be inferred from Equation 17.

$$\frac{dp_s}{dt} = \frac{dp_\phi}{dt} = ae^{-bt^2} \dots\dots\dots (16)$$

$$ae^{-bt^2} = -\frac{q_{gL}P_L}{V_{gs}} - \frac{p_s}{V_{gs}} \frac{dV_{gs}}{dt} \dots\dots\dots (17)$$

Two possible interpretations of this form are that  $V_{gs}$  can be factored (since it the only term in common) or that if  $p_L$  is proportional to  $p_s$ , that the ratio,  $V_{gs}/p_s$  can be factored. Assuming the second case, rewriting Equation 17 and expressing the gas volume in terms of the fluid level yields Equation 18.

$$ae^{-bt^2} = -\left(C_1q_{gL} + \frac{dx_L}{dt}\right) \frac{p_s}{x_L} \dots\dots\dots (18)$$

Comparing this with the form shown in Equation 16 indicates that one of the two terms must be constant (a), while the other must decrease exponentially with the square of time. Since Hegeman, et. al.<sup>3</sup> observed that a negative phase redistribution seems to apply at times, we infer that the term in parenthesis must be equivalent to a, since  $p_s/x_L$  cannot be negative. Therefore, the second term must be equivalent to the exponential, as shown in Equation 19.

$$a = -\left(C_1q_{gL} + \frac{dx_L}{dt}\right) \dots\dots\dots (19)$$

$$\frac{p_s}{x_L} = e^{-bt^2}$$

In this relation, it is apparent that when the rate of gas influx exceeds the rate of fluid level change, the positive phase redistribution will apply, which is apparently the normal case. If, however, the fluid level drops more rapidly than the gas influx rate, a negative value for a will result. This is consistent with the observation that negative phase redistribution may occur when gas goes into solution in the wellbore. Under those conditions, the gas volume disappears from the liquid column, thereby lowering the liquid level, without increasing the mass of gas near the top of the well.

Note also that this form predicts that the ratio between  $p_s$  and  $x_L$  will decrease rapidly. Therefore, if  $x_L$  approaches a constant value (a stabilized fluid level), then the surface pressure must drop continuously with time, approaching zero. Conversely, if a stable surface pressure is reached, then the fluid level must increase dramatically with time, which is non-physical. If, on the other hand, we assume that  $p_s$  cannot be factored from the terms in Equation 17, then it is apparent that the fluid level will increase even faster, which appears to be a worse condition.

In view of this analysis, it appears that the error function form for the phase redistribution function has a potential physical significance. When the surface pressure drops continuously during the shut-in, probably because of re-resolution of the gas in the liquid column, a static fluid level can result. But, if a static fluid level is approached, then the expression for  $a$  in Equation 18, indicates that the gas influx rate must increase, which is counter intuitive, unless the gas is continuously being dissolved in the liquid.

Therefore, it appears that the error function form for phase redistribution also suffers from a flaw in long time predicted performance. It may, however, be speculated that the error function is perhaps a limiting form when gas is being re-dissolved in the liquid column.

Now that we begin to understand the possible limiting forms for wellbore phase redistribution, it is of interest to evaluate the possibility of better formulations. It appears that both forms have appealing aspects, since the exponential gives a gas superficial velocity which depends somewhat consistently with density. The error function form, however, predicts a stable fluid level, which is also appealing. Both formulations, however, appear to suffer from non-physical fluid level behavior at long times.

This subject will be investigated in a future TechNote.

## **NOMENCLATURE**

$m_g$	mass of gas
$p_s$	surface pressure
$p_\phi$	phase redistribution pressure
$t$	time
$V_g$	volume of gas
$x_L$	fluid level (from surface)
$\rho_g$	mass of gas
$z$	real gas z-factor

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