# TechNotes March 1994


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# NOTES ON VARIABLE STORAGE IN PRESSURE TRANSIENT ANALYSIS


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In pressure transient analysis, it has been recognized for a long time that wellbore effects are important. The original analysis by Van Everdingen and Hurst[1] represented wellbore effects as a simple mass storage term they called "wellbore storage." Later Fair[2] added the description of phase redistribution to account for the gravity segregation of gas and liquid within the wellbore and defined a "phase redistribution function" of exponential form to account for pressure changes not due to the influx of fluids from the formation. Later Hegeman, et. al.[3] proposed an error function form for the phase redistribution pressure function to allow the matching of well tests where there appeared to be a change in the wellbore storage. Fair[4] later generalized the wellbore model to include the effect of momentum within the wellbore and showed that the so-called phase redistribution pressure function is actually the pressure change at constant wellbore mass and might be due to various phenomena including phase changes and thermal effects, although segregation of phases is likely the most common. In this note, the concept and implications of variable wellbore storage will be evaluated from a mathematical and physical viewpoint.

## *Mathematical Background*

The basic wellbore equation derived by Fair[4] is shown in equation 1, where momentum effects are neglected. It should be noted that the density ratio is essentially the formation volume factor. Note that if neither the bottomhole or phase redistribution pressures change, then their derivatives are zero and the surface and bottomhole flow rates are directly related to the density ratio or formation volume factor.

$$q_w = q_s\left(\frac{\rho_s}{\rho_w}\right) - C\left[\frac{dp_w}{dt} - \frac{dp_\phi}{dt}\right] \quad \text{.......... (1)}$$

Reference 4 also shows that the short time pressure solution, when wellbore effects dominate, is equal to Equation 2. This implies that the slope of a pressure vs. time plot will be as indicated in Equation 3.

$$\Delta p_w = \frac{\Delta t}{C} + \Delta p_\phi \quad \text{.......... (2)}$$

$$\frac{\Delta p_w}{\Delta t} = \frac{1}{C} + \frac{\Delta p_\phi}{\Delta t} \quad \text{.......... (3)}$$

As can be seen, if there is no phase redistribution, the slope of the pressure vs. time graph will be equal to the reciprocal of the storage constant. This observation is often used to estimate the storage constant from the pressure data. However, if is apparent that the slope can also be a constant when the storage, C, is constant and the phase redistribution pressure function is also linear, so that $\Delta p_{\varphi i}/\Delta t$ is constant. As defined by Fair[2], there then appears to be an “apparent” storage that is related to both the phase redistribution and the actual wellbore storage.

In the well testing literature (cf. Reference 3) there is a tendency to diagnose a changing wellbore storage based solely on the slope of the pressure vs. time graph. On a log-log plot, a constant storage will appear as a 45 degree line. When such a line appears, many interpreters appear to diagnose the effect as being due to changing wellbore storage. However, in light of the above equations, there may be a linear trend in pressure vs. time due to either a constant storage with no phase redistribution or due to a linear trend in phase redistribution with constant storage. It will be shown that the mathematical evaluation of these two cases are very different and to confuse them constitutes and error in pressure transient analysis.

## *Changing Storage*

First consider the case where the phase redistribution function is linear. In both the exponential and error function representations, early time behavior is approximately linear and an observed linear tendency in the pressure vs. time plot shows the combined effects of both storage and phase redistribution. The slope of the early time pressure behavior is equal to the apparent storage as defined in references 2 and 4. The appearance of two periods of linear behavior does not require the storage coefficient to change.

In the second case, mentioned by Hegeman, et. al.[3], there may indeed be a change in the wellbore storage constant, C, with or without phase redistribution effects. For example, Hegeman, et. al. evaluated the performance of gas wells, where the fluid density depends upon the changing wellbore pressure and therefore leads to the conclusion that the storage coefficient must be changing during the test. As shown by Fair[4], the storage coefficient, C, is defined in terms of fluid density as show in Equation 4.

$$C=\frac{1}{\rho_w}\left(\frac{\partial m}{\partial p_s}\right)=\frac{\partial V_w}{\partial p_s}+\frac{V_w}{\rho_w}\frac{\partial \rho_w}{\partial p_s} \quad \text{.......... (4)}$$

For a gas well, the wellbore volume should be constant, however the wellbore fluid density changes nearly directly proportional to the pressure. For the limiting case of an ideal gas, the storage coefficient is approximately as shown in equation 5.

$$C\approx\frac{V_w}{p_s} \quad \text{.......... (5)}$$

It is also possible to rearrange the wellbore equation to include the effect of phase redistribution in the storage coefficient, as shown in Equation 6, resulting in a variable apparent storage shown in Equation 7.

$$q_w=q_s\left(\frac{\rho_s}{\rho_w}\right)-C\left[1-\frac{dp_\phi}{dt}/\frac{dp_w}{dt}\right]\left(\frac{dp_w}{dt}\right) \quad \text{.......... (6)}$$

and

$$C_a=C\left[1-\frac{dp_\phi}{dt}/\frac{dp_w}{dt}\right]=C\left[1-\frac{dp_\phi}{dp_w}\right] \quad \text{.......... (7)}$$

In this representation it seems that the apparent storage is made to vary in accordance with the wellbore pressure through the $dp_\varphi/dp_w$ derivative, but the ability to represent phase redistribution has been sacrificed. In addition, it is not apparent that the parameters represent $p_\varphi$ are directly related to the physical phenomena that causes the changing storage, thereby making the evaluation difficult to validate.

## Conclusions

In conclusion, it appears that it is possible to use the phase redistribution pressure function to indirectly represent a case of changing wellbore storage, however, since the parameters involved are not directly related to the changing storage physical phenomenon, such use must be considered arbitrary and impossible to validate. In cases where such a formulation is required to match measured pressure data, the apparent storage defined in Equation 7 should be computed to ensure that the changes are indeed representative of the physical phenomenon that they are intended to represent.

It should be noted that the specific forms proposed for phase redistribution are empirical in nature as stated by both Fair2 and Hegeman, et. al.3. This is also shown in Reference 4, where the specific term representing phase segregation is actually the pressure change at constant wellbore mass and may represent various phenomena in addition to the segregation of phases.

## Nomenclature

| | |
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| $C$ | wellbore storage parameter |
| $C_\alpha$ | apparent wellbore storage parameter |
| $m$ | wellbore fluid mass |
| $p_{\varphi i}$ | phase redistibution pressure |
| $p_s$ | surface pressure |
| $p_w$ | bottomhole pressure |
| $q$ | volumetric flow rate |
| $t$ | time |
| $V$ | wellbore fluid volume |
| $\rho$ | density |

## References

1. Van Everdingen, A. F. and W. Hurst, “The Application of the Laplace Transformation to Flow Problems in Reservoir,” Trans., AIME (194) **186**, 305-324.

2 Fair, Walter B. Jr., “Pressure Buildup Analysis with Wellbore Phase Redistribution,” *SPEJ* (April 1981), 259-270.

3 Hegeman, P. S, D. L. Hallford and J. A Joseph, “Well Test Analysis with Changing Wellbore Storage,” SPE 21829, Rocky Mountain Regional Meeting, 1991.

4. Fair, Walter Jr., “Generalization of Wellbore Effects in Pressure Transient Analysis,” SPE 24715, Society of Petroleum Engineers, Washington, DC, 1992.